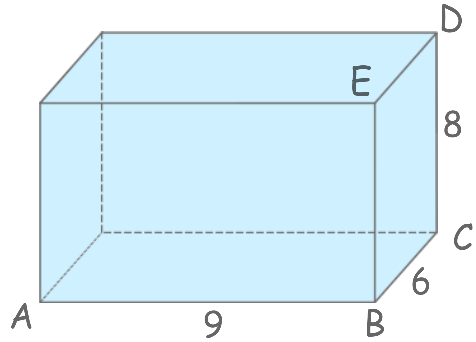
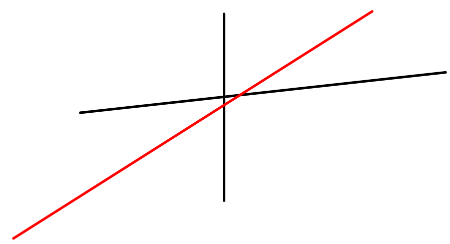
Precalc Warm Up # 13-3

1. Find $|\overrightarrow{AD}|$



2. If H is the midpoint of $\overrightarrow{DE}$, find $|\overrightarrow{AH}|$



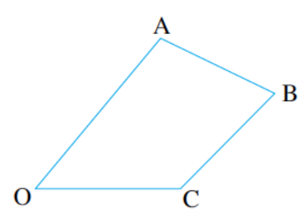
EXERCISES 12.3 p. 421

1. For the quadrilateral OABC, where **OA** = ***a***, **OB** = ***b*** and **OC** = ***c***, find in terms of ***a***, ***b*** and ***c*** an expression for

(a) **AC**

(b) **BC**

(c) the mid-point of $\overline{AB}$ relative to O



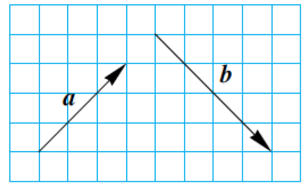
2. Using the vectors shown below, draw the vectors

(a) $\boldsymbol{a} - \boldsymbol{b}$

(b) $\boldsymbol{b} - 2\boldsymbol{a}$

(c) $2\boldsymbol{b} - 3\boldsymbol{a}$

(d) $\frac{1}{2}(\boldsymbol{b} + 2\boldsymbol{a})$



3. Simplify

(a) $\mathbf{WX} + \mathbf{XY} + \mathbf{YW}$

(b) $\mathbf{PQ} - \mathbf{SR} + \mathbf{QR}$

(c) $\mathbf{AX} - \mathbf{BX} + \mathbf{BZ} \cancel{+ \mathbf{YD}} - \mathbf{YZ} \cancel{- \mathbf{YD}}$

(d) $3\mathbf{OA} + 6\mathbf{BC} + 2\mathbf{AO} + \mathbf{AB} + 5\mathbf{OB}$

$-BX \rightarrow +XB$

So: $AX + XB + BZ + ZY$

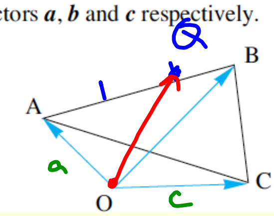
$= AY$

4. Consider the triangle ABC whose vertices have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively. Find the position vector of

(a) P, the mid-point of $\overline{AB}$.

(b) Q, the point of trisection of $\overline{AB}$, with Q closer to B.

(c) R, the mid-point of the median CP.



$$OA + \frac{2}{3}(AB)$$

$$a + \frac{2}{3}(-a+b)$$

$$\frac{1}{3}a + \frac{2}{3}b$$

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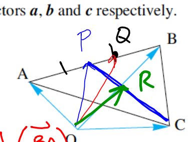


a) $\overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AB})$

$a + \frac{1}{2}(-a+b)$

$\frac{1}{2}a + \frac{1}{2}b$

b) $\overrightarrow{OB} + \frac{1}{3}(\overrightarrow{BA})$

$b + \frac{1}{3}(-\overrightarrow{AB})$

$b - \frac{1}{3}(-a+b)$

$b + \frac{1}{3}a - \frac{1}{3}b$

$\frac{2}{3}b + \frac{1}{3}a$

c) $\overrightarrow{OR} = \overrightarrow{OP} + \frac{1}{2}(\overrightarrow{PC})$

$= \frac{a+b}{2} + \frac{1}{2}(\overrightarrow{PO} + \overrightarrow{OC})$

$= \frac{a+b}{2} + \frac{1}{2}\left(-\frac{a+b}{2} + c\right)$

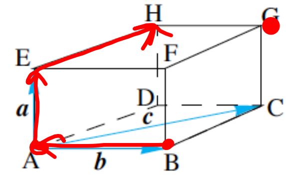
$= \frac{(a+b)}{2}\cdot\frac{2}{2} - \frac{a+b}{4} + \frac{c}{2}\cdot\frac{2}{2}$

$= \frac{2a+2b-a-b+2c}{4}$

$= \frac{a+b+2c}{4}$

7. Consider the cuboid ABCD, EFGH with vectors as shown on the diagram. Express, in terms of ***a***, ***b*** and ***c*** the following:

(a) **BC**
(b) **AG**
(c) **BH**



8. In question 7., $|\boldsymbol{a}| = 2$, $|\boldsymbol{b}| = 4$ and $|\boldsymbol{c}| = 10$. Find (a) BC (b) BH

$-\vec{b} + \vec{a} + \vec{EH}$

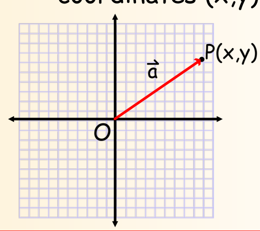
$\vec{EG} + \vec{GH}$

$-\vec{b} + \vec{a} + \vec{c} - \vec{b}$

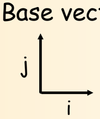
We can express vectors as column vectors which can make working with them easier.

$\vec{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ is the position vector $\vec{OP}$ where P has the coordinates (x,y)

$\vec{a} = \vec{OP}$



Base vector notation:



$\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{matrix} x \\ y \end{matrix}$ $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{matrix} x \\ y \end{matrix}$

Vector $\boxed{\vec{a} = 2i - j}$ is called a "component vector".
Write this as a column vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

What is $|\vec{a}|$?

$\sqrt{2^2 + (-1)^2}$

$\sqrt{5}$

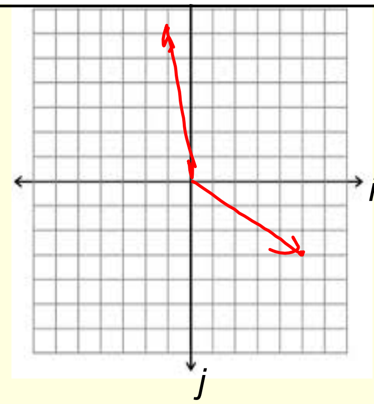
Graph

1. 5i-3j

2. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ $\sqrt{37}$

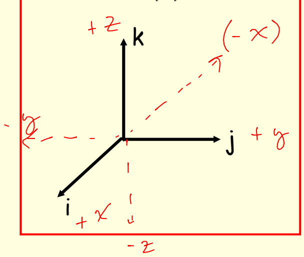
3. Find the magnitude of each vector above.

$\sqrt{34}$



We often represent vectors in 3-D

i= $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ j= $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ k= $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

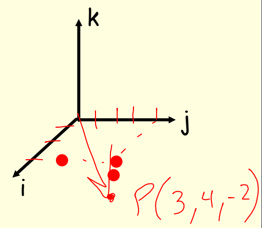


Graph the vector, and find its magnitude

$\vec{p} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

$\sqrt{9+16+4}$

$\sqrt{29}$



$\vec{s} = 2\begin{pmatrix} -1 \\ 4 \\ 6 \end{pmatrix}$ $-\vec{p} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$

Find $2\vec{s} - \vec{p}$

a) by using column vectors:

$\begin{pmatrix} -2 \\ 8 \\ 12 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 12 \\ 9 \end{pmatrix}$

b) by writing $\vec{s}$ and $\vec{p}$ as component vectors:

$2(-i+4j+6k)$

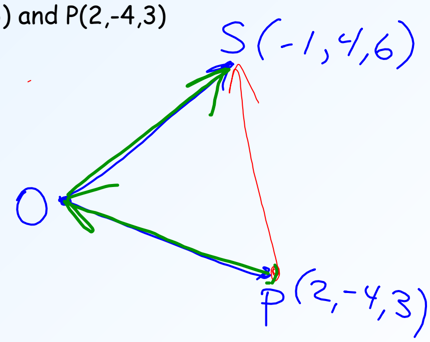
$-(2i-4j+3k)$

s and p are considered position vectors that join the origin to the points S(-1,4,6) and P(2,-4,3)

Find $\overrightarrow{PS}$

$\overrightarrow{PO} + \overrightarrow{OS}$

$\begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \\ 6 \end{pmatrix}$



Find the magnitude of each vector. It's not necessary to simplify these "surds".

1. 2i - 6j

$\sqrt{40}$

2. i - j + 4k

$\sqrt{18}$

3. $p = \frac{1}{\sqrt{29}}\begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$

$\sqrt{29}$

A unit vector is 1 long. Are any of the above vectors unit vectors? How could we make them unit vectors?

$\frac{1}{\sqrt{40}}(2i - 6j)$ $\frac{1}{\sqrt{18}}(i - j + 4k)$

If $\vec{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ = xi + yj + zk, $|a| = \sqrt{x^2 + y^2 + z^2}$

the unit vector is $\hat{a} = \frac{a}{|a|}$ → a "hat"

length = 1

Find a unit vector in the same directions as the vectors:

a. i - 3j + 2k

$\sqrt{14}$ $\frac{1}{\sqrt{14}}(i - 3j + 2k)$

b. $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\frac{1}{\sqrt{17}}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

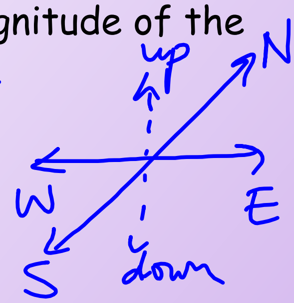
$\sqrt{17}$

Find a vector of length 7 in the direction of i + 2j - 3k

$$\frac{7}{\sqrt{14}}(i+2j-3k)$$

If something is being pulled by a force of 5 Newtons in a North direction, 6 Newtons in a East direction and 2 Newtons down, what is the total magnitude of the force acting on the object?

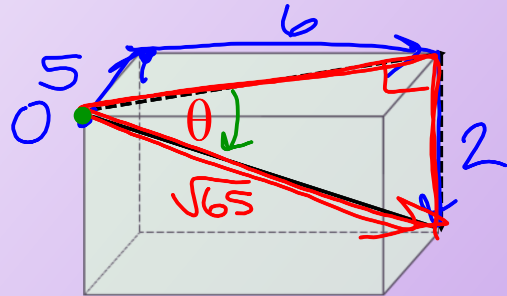
$$\begin{array}{c} x \\ y \\ z \end{array} \begin{pmatrix} \text{east} \\ \text{north} \\ \text{up} \end{pmatrix} \begin{array}{c} i \\ j \\ k \end{array}$$



$$\begin{array}{l} \text{6 East: } + \\ \text{5 North: } + \\ \text{2 Down: } - \end{array} \begin{pmatrix} 6 \\ 5 \\ -2 \end{pmatrix}$$

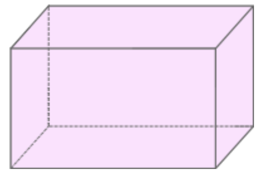
$$\sqrt{36+25+4}$$

What is the angle of depression, θ ?

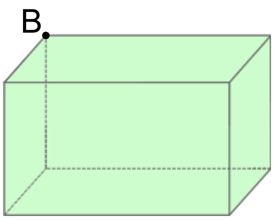


A boat (considered to be the origin of a vector system) is searching for sunken treasure. The treasure is believed to be 320 meters below the water and is 100 m east and 40 meters south of the boat. A helicopter is 1000 m west and 1500 m above the boat.

a. use $\begin{pmatrix} east \\ north \\ up \end{pmatrix}$ as suitable vector basis for this, and state the position vectors of the treasure and the helicopter



b. Write a vector to describe the path from the helicopter to the treasure. Find how far away the treasure is and what direction the treasure is from the helicopter.



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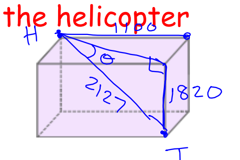


$$\overrightarrow{HT} = \overrightarrow{HB} + \overrightarrow{BT}$$

$$\begin{pmatrix}1000\\0\\-1500\end{pmatrix} + \begin{pmatrix}100\\-40\\-320\end{pmatrix}$$

H 1000

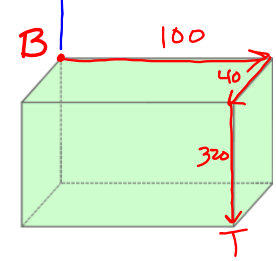
1500

$\sin\theta = \frac{1820}{2127}$

b. Write a vector to describe the path from the helicopter to the treasure. Find how far away the treasure is and what direction the treasure is from the helicopter.

$\theta \approx 58.8^\circ$

$$\begin{pmatrix}1100\\-40\\-1820\end{pmatrix}$$



$$\sqrt{1100^2 + (-40)^2 + (-1820)^2}$$

$$\approx 2127 \text{ m}$$

HW:

p. 427 #1-3 (I and IV only on these)

and 4, 5, 6 (i only), 7 (i only), 8-10

p. 431 #1 (i and viii only), 2 (i and viii only),

#3-5 (4 is wrong in the back)